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Discussion on the convective heat transfer and field synergy principle

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Abstract

The convective "heat" transfer is actually mainly carried out by the motion of hotter or colder particles from one system into another system. Therefore, the best convective "heat" (strictly speaking, internal energy) transfer is the case where velocity vectors are always perpendicular to the isothermal surfaces (or isotherm in 2D cases). This conclusion has been named "field synergy principle". In this paper, some field synergy exact solutions are presented to further develop the principle. The concrete physical meanings of the derived analytical solutions are analyzed. The method of separating variables with addition and other extraordinary approaches are adopted in the derivation.

Keywords: Convection; Field synergy; Analytical solution

1. Field synergy principle of convection and analytical solutions

"There are three different modes of heat transfer: conduction, convection and radiation". Such thinking has been established long time ago. However, strictly speaking, convection is not a kind of "heat" transfer according to classical thermodynamics [1]. And according to the ASME Heat Transfer Handbook [2], "it is not heat that is being convected, but internal energy" in convection processes. These concepts are useful to concisely prove and quantitatively indicate the "field synergy principle".

Since the convection is mainly put in practice by the movement of hotter or colder particles carrying higher or lower internal energy, the directions of velocity vectors are very important. If the movement directions of all particles were completely following the isothermal lines (2D case) or the isothermal surfaces (3D case), it could be concluded that no convection effects would occur. And the heat transfer would be very poor, near to adiabatic if neglecting heat conduction. Conversely, the convection result would be the best if the movement directions of all particles were completely perpendicular to the isothermal lines (2D case) or the isothermal surfaces (3D case). Such conclusion is arrived only by physical thinking. The strict mathematical derivation of this concept has been given and improved by Guo and Tao, etc. [3–5] since the end of last century. They called it "field synergy principle" of convection. Recently, they confirmed this principle with many numerical and experimental studies [6–11]. In addition, they applied this principle to improve some heat transfer apparatuses and obtained excellent results.

In this paper, several algebraically explicit incompressible 2D exact analytical solutions are derived. Some solutions are full field synergy ones, in which velocity vectors are always perpendicular to the isothermal lines. Some solutions are boundary field synergy ones, in which synergy occurs only along the boundary between fluid and its container. They are meaningful to verify the possibility of the existence of full field synergy and to further develop the field synergy principle. In addition, it is well known that the analytical exact solutions have their own theoretical meaning. Many analytical solutions played a key role in the early development of fluid mechanics and heat conduction [12,13]. Besides their theoretical meaning, analytical

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Nomenclature

a	thermal diffusivity	β	coefficient of expansion
c	constant	θ	temperature
$f(x)$	function of x	ν	kinematic viscosity
g	acceleration of gravity	ρ	density
G	mass source		
$g(y)$	function of y	*Subscripts*	
p	pressure	G	function for mass source
q	heat source	p	function for pressure
u	velocity component in x direction	q	function for density
υ	velocity component in y direction	u	function for velocity component u
X	function of x	υ	function for velocity component υ
x	abscissa	θ	function for temperature
Y	function of y	0, 1, 2, 3 ...	different constants
y	ordinate	∞	constants

solutions can also be applied to check the accuracy, convergence and effectiveness of various numerical computation methods and to improve their differencing schemes, grid generation ways and so on. The analytical solutions are therefore very useful even for the newly rapidly developing computational fluid dynamics and heat transfer. For example, several analytical solutions that can simulate the 3D potential flow in turbomachine cascades were obtained by Cai et al. [14]. And they were successfully utilized by some investigators in their numerical calculation to check their computational techniques and computer codes [14–17].

2. Governing equation set and analytical solution derivation

The governing equation set of steady 2D incompressible laminar flow with constant kinematic viscosity ν and thermal diffusivity a (neglecting gravity and dissipation heat) can be presented as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial \upsilon}{\partial y}=G, \tag{1}$$

$$u\frac{\partial u}{\partial x}+\upsilon\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right), \tag{2}$$

$$u\frac{\partial \upsilon}{\partial x}+\upsilon\frac{\partial \upsilon}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\left(\frac{\partial^2 \upsilon}{\partial x^2}+\frac{\partial^2 \upsilon}{\partial y^2}\right), \tag{3}$$

$$u\frac{\partial \theta}{\partial x}+\upsilon\frac{\partial \theta}{\partial y}=a\left(\frac{\partial^2 \theta}{\partial x^2}+\frac{\partial^2 \theta}{\partial y^2}\right)+q. \tag{4}$$

Considering the requirement of full field synergy, following equation needs to be complemented.

$$u/\upsilon=\frac{\partial \theta}{\partial x}\bigg/\frac{\partial \theta}{\partial y}. \tag{5}$$

Heat source q and mass flow source G are commonly given functions. There are only four unknown variables in the above-mentioned five equations, namely velocity components u and υ, pressure p and temperature θ. Then the equation number is more than the number of independent variables. The problem is not properly posed and generally unable to obtain solutions. Indeed, no meaningful solutions have been derived yet for the above-mentioned equation set without heat or mass flow sources except for even temperature condition. Therefore q or G needs to be an unknown variable to satisfy the number of equations. Actually, it physically means that it is very difficult to find a fully field synergy condition without any artificial measures. In other words, it is possible to obtain field synergy cases with some measures to control the field, for example, adding appropriate heat source or mass flow source. Therefore, q or G in the governing equation set has to be recognized as a control measure and an important variable.

Governing equation set (1)–(5) are nonlinear simultaneous partial differential equations, not easy to be solved. In order to obtain algebraically explicit exact analytical solutions for evidently understanding the results, the method of separating variables with addition promoted by the first author [18,19] is applied. It is assumed that the unknown solution has the form of $f(x,y)=X(x)+Y(y)$ rather than $f(x,y)=X(x)\cdot Y(y)$ in the common method of separating variables. The main aim here is to obtain some possible field synergy explicit analytical solutions to develop the theory and promote computational heat transfer (CHT) but not to find a specified solution for given boundary conditions. Therefore the boundary conditions are undetermined before derivation and deduced from the solution afterward. It makes the derivation procedure easier. Indeed, sometimes the derivation procedure is basically a trial and error one with the help of inspiration, experience and fortune. The above-mentioned approaches have been successfully applied to derive many meaningful algebraically explicit analytical solutions of heat and mass transfer discipline [18–32].

Actually, all solutions given in this paper can be proven easily by substituting them into the governing equation set.

Applying the method of separating variables with addition to all variables in Eqs. (1)–(5), it is assumed that:

$$u = X_u + Y_u, \quad (6)$$
$$\upsilon = X_\upsilon + Y_\upsilon, \quad (7)$$
$$G = X_G + Y_G, \quad (8)$$
$$p = X_p + Y_p, \quad (9)$$
$$\theta = X_\theta + Y_\theta, \quad (10)$$

and

$$q = X_q + Y_q. \quad (11)$$

Then Eqs. (1)–(5) can be changed into:

$$X'_u + Y'_\upsilon = X_G + Y_G, \quad (12)$$
$$(X_u + Y_u)X'_u + (X_\upsilon + Y_\upsilon)Y'_u = -X'_p/\rho + \nu(X''_u + Y''_u), \quad (13)$$
$$(X_u + Y_u)X'_\upsilon + (X_\upsilon + Y_\upsilon)Y'_\upsilon = -Y'_p/\rho + \nu(X''_\upsilon + Y''_\upsilon), \quad (14)$$
$$(X_u + Y_u)X'_\theta + (X_\upsilon + Y_\upsilon)Y'_\theta = a(X''_\theta + Y''_\theta) + X_q + Y_q, \quad (15)$$

and

$$(X_u + Y_u)Y'_\theta = (X_\upsilon + Y_\upsilon)X'_\theta. \quad (16)$$

In the following derivation procedure, the above-mentioned equations (12)–(16) are frequently applied to obtain algebraically explicit exact solutions for field synergy condition. In addition, a hybrid approach with both separating method is also applied in this paper.

3. Analytical full field synergy solution with heat source (I) – using the method of separating all variables with addition

To control the 2D heat transfer field, a distributed heat source is possible to put in practice. For example, sometimes it can be done by radiation. A simple synergy solution with only heat source is first derived. Its simplified form with very clear physical meaning is given in the next paragraph.

The method of separating variables with addition is adopted. The governing equation set becomes Eqs. (12)–(16) with $X_G = 0 = Y_G$.

For such case, Eq. (12) can be separated easily and the result is:

$$X'_u = c_1 = -Y'_\upsilon, \quad (17)$$

It is derived

$$X_u = c_1 x + c_2, \quad (18)$$

and

$$Y_\upsilon = c_3 - c_1 y. \quad (19)$$

If $X_\upsilon = c_4$, Eq. (13) can be separated as

$$c_1^2 x + c_1 c_2 + X'_p/\rho = c_5 = -c_1 Y_u - (c_3 + c_4)Y'_u + c_1 y Y'_u + \nu Y''_u. \quad (20)$$

The right side of Eq. (20) can be analytically solved only when $c_1 = 0$. Since the aim of this paper is to find analytical exact solutions, $c_1 = 0$ is assumed. Then from the left side and the right side of Eq. (20), following results can be deduced:

$$X_p = p_0 + c_5 \rho x, \quad (21)$$

and

$$Y_u = [\nu/(c_3 + c_4)]^2 \exp[(c_3 + c_4)(y + c_6)/\nu] - c_5 y/(c_3 + c_4). \quad (22)$$

Since $X'_u = Y'_\upsilon = 0$, from Eq. (14) it is obtained $Y'_p = 0$. It means Y_p is a constant. According to Eqs. (9) and (21), it can be regarded as zero:

$$Y_p = 0. \quad (23)$$

Substituting above results (including $c_1 = 0$) into Eq. (16), following separated equation is obtained:

$$\{[\nu/(c_3 + c_4)]^2 \exp[(c_3 + c_4)(y + c_6)/\nu] - c_5 y/(c_3 + c_4) + c_2\}Y'_\theta = c_7 = (c_3 + c_4)X'_\theta. \quad (24)$$

The left side of Eq. (24) can be only analytically solved when $c_2 = 0 = c_5$. Using this simplification, the following two solutions are derived

$$Y_\theta = -c_3 c_7 \exp[c_3(y + c_6)/\nu]/\nu, \quad (25)$$
$$X_\theta = \theta_0 + c_7 x/c_3. \quad (26)$$

Substituting the above solutions into Eq. (15), the heat source q can be derived as

$$q = c_7[g(y) + (1 + a/\nu)/g(y)], \quad (27)$$

where

$$g(y) = \nu^2 \exp[c_3(y + c_6)/\nu]/c_3^3 \quad (28)$$

Combining Eqs. (12)–(16) with all previous results in this paragraph, the final solution can be expressed as follows. Because $c_3 + c_4$ always appears together in the final result, c_3 is chosen on behalf of $c_3 + c_4$.

$$u = (\nu/c_3)^2 \exp[c_3(y + c_6)/\nu] = c_3 g(y), \quad (29)$$
$$\upsilon = c_3, \quad (30)$$
$$p = p_0, \quad (31)$$
$$\theta = \theta_0 + c_7 x/c_3 - c_3 c_7 \exp[-c_3(y + c_6)/\nu]/\nu = \theta_0 + c_7 x/c_3 - c_7 \nu/[c_3^2 g(y)] \quad (32)$$
$$q = c_7[g(y) + (1 + a/\nu)/g(y)]. \quad (33)$$

The function $g(y)$ is given in Eq. (28).

As mentioned before, the boundary conditions are determined after successfully deriving the solution. The conditions can be obtained by substituting the geometries of the boundaries into the solution. For example, if considering the boundary was a rectangle with unity width, the boundary conditions of the solution in this paragraph could be:

$y = 0$:

$$u = (\nu/c_3)^2 \exp(c_3 c_6/\nu), \quad \theta = \theta_0 + c_7 x/c_3 - c_3 c_7 \cdot \exp(-c_3 c_6/\nu)/\nu;$$

and

$y = 1$:

$$u = (v/c_3)^2 \exp[c_3(1+c_6)/v],$$
$$\theta = \theta_0 + c_7 x/c_3 - c_3 c_7 \cdot \exp[-c_3(1+c_6)/v]/v;$$

in addition,

$x = 0$:

$$u = (v/c_3)^2 \exp[c_3(y+c_6)/v],$$
$$\theta = \theta_0 - c_3 c_7 \cdot \exp[-c_3(y+c_6)/v]/v,$$

$x = 1$:

$$u = (v/c_3)^2 \exp[c_3(y+c_6)/v],$$
$$\theta = \theta_0 + c_7/c_3 - c_3 c_7 \exp[-c_3(y+c_6)/v]/v.$$

In the whole field there are $\upsilon = c_3$ and $p = p_0$. The q distribution can be recognized as the source, being excluded from the boundary conditions. The boundary conditions of other solutions given in the following paragraphs can be determined similarly. Each solution corresponds to its own boundary conditions.

The physical description of the solution with constants $c_3 < 0$, $c_6 = 0$ and $c_7 > 0$ is shown in Figs. 1 and 2. The first one presents the flow between two infinite porous plates parallel to x abscissa moving along the abscissa direction with different speeds. Their speeds are given by Eq. (29) with $y = 0$ and $y = 1$ to satisfy the no slip condition in viscous flow. The flow field between the porous plates described by Eqs. (29) and (30) is a synergy field. The x-direction speed u is a 1D exponential function of y. The y-direction speed υ is a constant c_3 in the whole field including in the porous plates. The temperature distribution is a 2D function: linear along x-direction and exponential in y-direction. The isothermal lines have to be completely perpendicular to the stream lines. The expression of the stream lines can be derived by $dy/dx = \upsilon/u$ and the result is $x = \{(v/c_3)^3 \exp[c_3(y+c_6)/v] - c_8\}/c_3$. Both stream lines and isothermal lines in the considered field are shown in Fig. 2. The heat source distribution is a 1D exponential function of y.

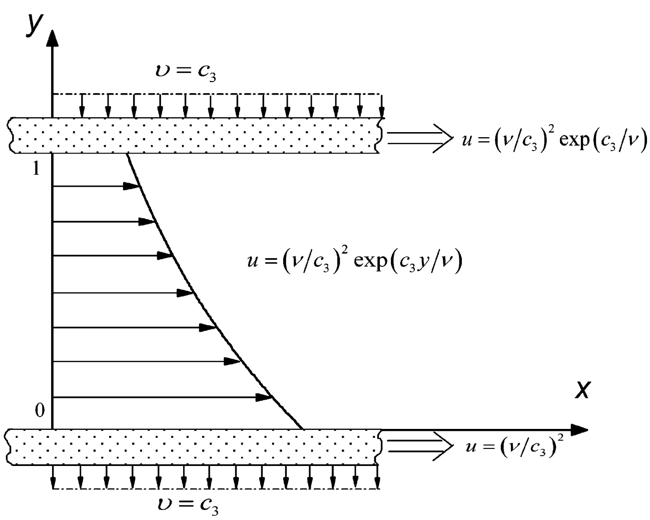

Fig. 1. The flow condition of Eqs. (29)–(33) with $c_3 < 0$, $c_6 = 0$ and $c_7 > 0$.

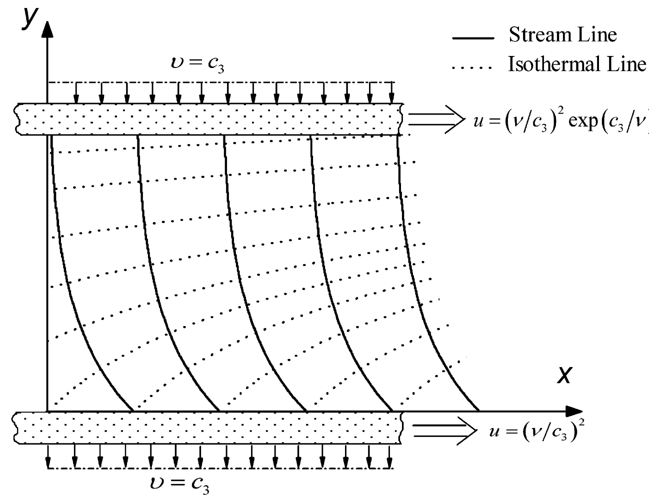

Fig. 2. The stream lines and isothermal lines of Eqs. (29)–(33) with $c_3 < 0$, $c_6 = 0$ and $c_7 > 0$.

4. Analytical full field synergy solutions with heat source (II) – concise solution family using the method of separating variables with addition

The physical feature of the solution in the previous paragraph is a little bit complicated (with two parallel moving porous walls as boundary). In this paragraph, a very simple solution family with two infinite parallel steady solid walls as boundary is given.

The simplest way to satisfy the no slip condition on the parallel solid tunnel walls is

$$u = c_1 y^2 + c_2, \tag{34}$$

and

$$\upsilon = 0. \tag{35}$$

It corresponds to the following assumptions in Eqs. (6)–(16), and satisfies Eq. (12) with $X_G = 0 = Y_G$.

$$X_u = c_2, \tag{36}$$
$$Y_u = c_1 y^2, \tag{37}$$
$$X_\upsilon = Y_\upsilon = 0, \tag{38}$$

With these assumptions, it is easy to obtain following result from the momentum equations (13) and (14):

$$X_p = 2c_1 \rho v x, \tag{39}$$
$$Y_p = p_0. \tag{40}$$

Then the pressure formula is

$$p = p_0 + 2c_1 \rho v x. \tag{41}$$

Since $\upsilon = 0$, it is deduced from Eq. (16) $Y'_\theta = 0$. In other words Y_θ is a constant. Using the same equation, it is concluded that X_θ can be an arbitrary function of x, which means

$$\theta = \text{arbitrary} f(x). \tag{42}$$

Finally, the formula of heat source q (Eq. (15)) can be easily solved as

$$q = (c_1 y^2 + c_2) f'(x) - a f''(x). \tag{43}$$

Eqs. (34), (35) and (41)–(43) represent a family of simple field synergy solutions. The number of the solutions in the family is infinite since there is an arbitrary function $f(x)$. However, the velocity distribution – parabolic curve along y-direction – is the same for the whole solution family, similar to the classical 2D Poiseuille flow. The pressure distribution is linear along x-direction similar to Poiseuille flow as well. The main distinguishing feature is the heat source distribution. It controls the distribution of temperature and guarantees the field synergy. The heat source function has evident relationship with the temperature distribution (Eqs. (42) and (43)). Among the variables, flow velocity is a function of y; thermodynamic parameters – pressure and temperature are functions of x; heat source q is a 2D function.

Next we analyze some representative functions of $f(x)$ and their features.

4.1. Solution with $f(x)$ = Const

If $f(x) = \text{Const}$, there is $\theta = \text{Const}$ and $q = 0$; no heat transfer occurs. The solution approximates to Poiseuille flow and is not meaningful for field synergy principle.

4.2. Solution with linear temperature distribution

If $f(x)$ is a linear function, for example $f(x) = c_3 x + c_4$, then

$$\theta = c_3 x + c_4, \tag{44}$$

and

$$q = c_3 (c_1 y^2 + c_2). \tag{45}$$

Besides the velocity and pressure distributions, it means the temperature is a 1D linear function of x. The isothermal lines are simple vertical lines. Because there is only x-direction velocity u, the field synergy condition is evidently fulfilled. The heat source distribution is similar to the velocity distribution. They are all 1D function of y.

The feature of this solution with $c_1 < 0$, $c_2 = -c_1$ and $c_3 = 1$ is given in Fig. 3. It is a very simple and clear field synergy flow.

In addition, another very simple solution similar to Fig. 3 with even isothermal line but with constant heat source distribution can be derived as a simplified case of the solution given by Eqs. (34), (35) and (41)–(43). When $c_1 = 0$, $f(x) = c_2 x$, the simplified solution is

$$\left.\begin{array}{ll} u = c_2 = \text{Const} & v = 0 \\ p = p_0 = \text{Const} & \theta = c_3 x + c_4 \\ q = c_2 c_3 = \text{Const} & \end{array}\right\} \tag{46}$$

The feature of this solution is given in Fig. 4. If considering a nonviscous flow, both channel walls can be regarded steady. But for viscous flow, the walls have to move towards right with the same velocity of the fluid – u to satisfy the no slip condition. The distinguishing feature of this solution is that the heat source is constant. Therefore it is very easy to put in practice.

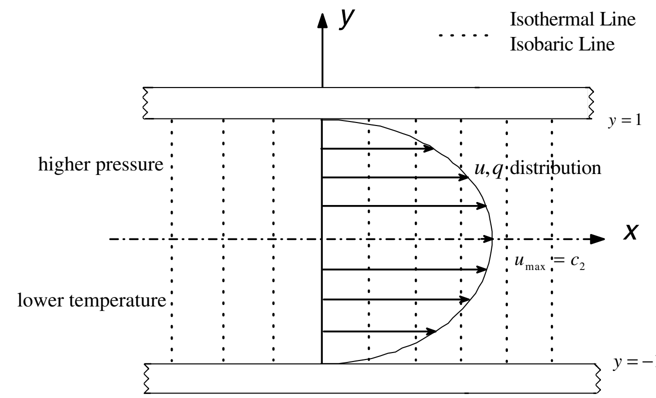

Fig. 3. The flow condition of the solution in Section 4.2 with $c_1 < 0$, $c_2 = -c_1$ and $c_3 = 1$.

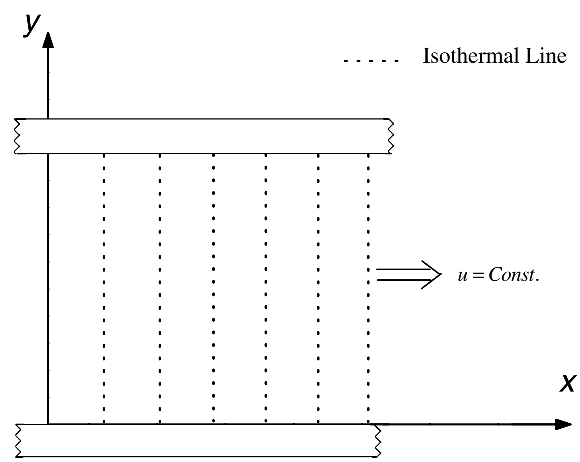

Fig. 4. Field synergy with a constant heat source.

4.3. Full field synergy solution without sources

For above-mentioned Poiseuille flow, can we find the field synergy solutions without mass and heat sources? If assuming $q = 0$, Eq. (43) can be expressed as

$$(c_1 y^2 + c_2)/a = f''(x)/f'(x). \tag{47}$$

Eq. (47) can be solved only when $c_1 = 0$. Then the equation becomes an ordinary differential equation

$$f''(x) = c_2 f'(x)/a. \tag{48}$$

The solution is

$$f(x) = \theta = c_5 a \cdot \exp(c_2 x/a)/c_2. \tag{49}$$

However, according to Eqs. (34) and (41), $c_1 = 0$ means velocity is a constant $u = c_2$ and pressure is a constant as well. The viscosity v does not appear in the equations. Therefore the solution is for the ideal nonviscous flow

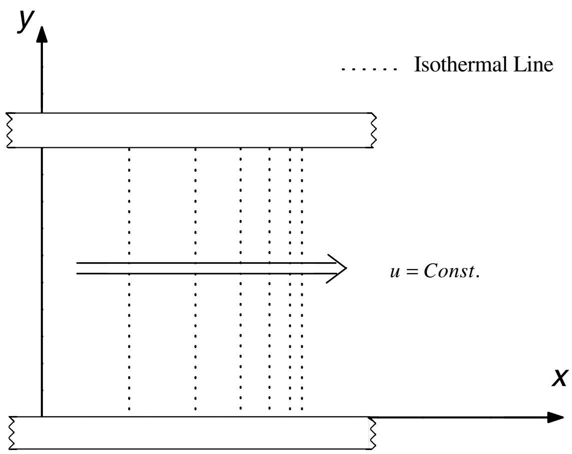

Fig. 5. An ideal case of field synergy.

without boundary layer. A simplified illustration of this solution is given in Fig. 5. The velocity vector is constant and horizontal. The isothermal lines with the same temperature difference are vertical and located denser and denser from left to right. A similar solution and figure were given some years ago [4] without mathematic derivation and the distance between the isothermal lines is even in that figure. It seems such figure only mentioned the perpendicular relationship between velocity and isothermal lines but neglected the arrangement of isothermal lines. By the way, a linear function $\theta(x)$ cannot satisfy the governing equation (4) with $u = \text{Const}$, $q = 0$ and $v = 0$.

In summary, we can find field synergy in the steady wall Poiseuille flow without mass and heat sources for only ideal nonviscous case.

By the way, if both walls of the flow channel are considered moving towards right with the same velocity of the fluid – u – and satisfying the no slip condition, it can be recognized as a viscous flow.

4.4. Other possible solutions

Using Eqs. (34), (35) and (41)–(43) and choosing different $\theta = f(x)$, infinite field synergy solutions can be derived easily. Their velocity and pressure distributions are similar to that of Poiseuille flow. Only the temperature distributions $\theta(x)$ and corresponding heat source distributions are different. By the way, we have not yet found any other solutions with clear physical characteristics as Figs. 3–5. However, perhaps new solutions could be found later. The solutions given in this paragraph are based on Poiseuille flow, which is a very popular case.

5. Analytical full field synergy solution with heat source (III) – using the hybrid method of separating variables

The method of separating variables with addition has been indeed successfully applied to derive many analytical solutions. However, for simultaneous equations with two or more variables, it is not necessary to apply the same separating approach for all variables. Hybrid methods can be used. For example, some variables could be treated with the method of separating variables with addition, and the others could be treated with the common method of separating variables with multiplication. In this paragraph, instead of Eq. (10), it is assumed that

$$\theta(x,y) = X_\theta \cdot Y_\theta. \tag{50}$$

But other variables are assumed as the same as before Eqs. (6)–(9).

In this case, the Eqs. (12)–(14) are still effective. But the Eq. (16) has to be changed into

$$(X_u + Y_u)X_\theta Y'_\theta = (X_v + Y_v)X'_\theta Y_\theta. \tag{51}$$

Then the velocity and pressure distributions are the same with Eqs. (29)–(31). But the temperature distribution should be derived from Eq. (51) with known velocity distribution equations (29) and (30). The expression is

$$(v/c_3)^2 \exp[c_3(y + c_6)/v]X_\theta Y'_\theta = c_3 X'_\theta Y_\theta. \tag{52}$$

After separating variables, following two ordinary differential equations are obtained:

$$(v^2/c_3^3)\exp[c_3(y + c_6)/v]Y'_\theta/Y_\theta = c_7 = X'_\theta/X_\theta. \tag{53}$$

The final result of Eqs. (50)–(53) is

$$\theta = \theta_0 + c_8 \exp(c_7 x) \cdot \exp\{-c_7 c_3^2 \exp[-c_3(y + c_6)/v]/v\}. \tag{54}$$

The heat source q can be derived easily by substituting the expressions of u, v and θ (Eqs. (25), (26) and (54)) into Eq. (4).

The physical feature of this solution is very similar to these of Figs. 1 and 2 in Section 3. But the heat source q here is a 2D function. The graphical expressions are not given here to shorten the space of the paper.

Besides obtaining an exact solution, another more meaningful result of this paragraph is the application of the new method of separating variables for partial differential equations. It should be developed further.

6. Analytical full field synergy solutions with mass source – using the method of separating variables with addition

All the solutions given in the above-mentioned three paragraphs only apply the heat source to achieve field synergy. In this paragraph, solutions utilizing only mass sources to achieve field synergy are derived. However, utilizing pure mass sources is commonly more complicated in practice compared with heat sources. For example, the temperature of each particle of the mass source has to be the same with the temperature at the injecting positions. Otherwise it is difficult to accurately control the temperature field. In addition, the particle motion would commonly disturb the flow field. Nevertheless, deriving some analytical solution is helpful for developing field synergy principle and understanding how to promote the field synergy.

In following derivation, $q = 0$ and $G \neq 0$ is adopted in the governing equation set.

6.1. The first solution

Omitting the trial and error procedure, the brief derivation is summarized as follows. Assuming

$$Y_u = \text{Const} = c_1, \tag{55}$$

$$X_\upsilon = \text{Const} = c_2. \tag{56}$$

Then the governing equations (13)–(16) become

$$(X_u + c_1)X_u' = -\frac{1}{\rho}X_p' + \nu X_u'', \tag{57}$$

$$(Y_\upsilon + c_2)Y_\upsilon' = -\frac{1}{\rho}Y_p' + \nu Y_\upsilon'', \tag{58}$$

$$(X_u + c_1)X_\theta' + (Y_\upsilon + c_2)Y_\theta' = a(X_\theta'' + Y_\theta'') \tag{59}$$

and

$$(X_u + c_1)/(Y_\upsilon + c_2) = X_\theta'/Y_\theta'. \tag{60}$$

After separating variables, they appear as:

$$X_p' = \rho[-(X_u + c_1)X_u' + \nu X_u''], \tag{61}$$

$$Y_p' = \rho[-(Y_\upsilon + c_2)Y_\upsilon' + \nu Y_u''], \tag{62}$$

$$aX_\theta'' - (X_u + c_1)X_\theta' = -c_3 = (Y_\upsilon + c_2)Y_\theta' - aY_\theta'' \tag{63}$$

and

$$(X_u + c_1)/X_\theta' = c_4 = (Y_\upsilon + c_2)/Y_\theta'. \tag{64}$$

From Eqs. (63) and (64) following results can be deduced:

$$\begin{aligned}\theta &= X_\theta + Y_\theta \\ &= \theta_0 + \sqrt{c_3/c_4}(x + c_6) - a\ln\{\cos[(\sqrt{c_3c_4}/a)(y - c_5)]\}/c_4.\end{aligned} \tag{65}$$

Then the velocities can be derived from Eq. (60) as:

$$u = \sqrt{c_3c_4}, \tag{66}$$

$$\upsilon = \sqrt{c_3c_4}\tan[\sqrt{c_3c_4}(y - c_5)/a]; \tag{67}$$

And the pressure expression is obtained as following according to Eqs. (61) and (62)

$$p = p_0 + c_3c_4\rho[(2\nu/a - 1)]\tan^2[\sqrt{c_3c_4}(y - c_5)/a]/2 \tag{68}$$

In addition, the mass source G is derived from Eq. (1) as:

$$G = c_3c_4\sec^2[\sqrt{c_3c_4}(y - c_5)/a]/a. \tag{69}$$

And the stream line equation is $x = x_0 + a\ln\{\sin[\sqrt{c_3c_4}(y - c_5)/a]\}/\sqrt{c_3c_4}$.

It can be found from the above-mentioned results that in this solution only temperature θ is a 2D function. Other parameters are functions of y(υ, p and G) or even a constant (u).

The physical description of the solution with $c_5 = 0$ is shown in Figs. 6 and 7. The former presents the flow between two infinite plates parallel to x abscissa moving along the abscissa direction with the same speed. The lower plate is a solid one but the upper is a porous one. The x-direction speed u in the channel is a constant and equal to the moving speed of the plates. The y-direction speed υ in the channel increases along y-direction from zero at lower plate. Finally the working fluid ejects out through the upper porous plate. The stream lines are shown in Fig. 7.

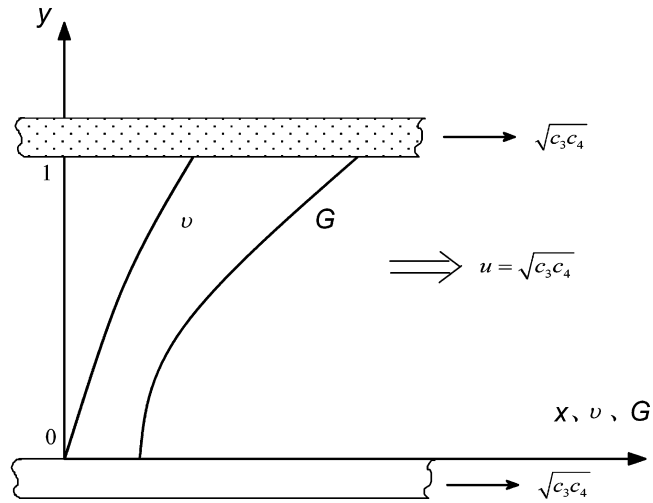

Fig. 6. The flow condition of Eqs. (66)–(69) with $c_5 = 0$.

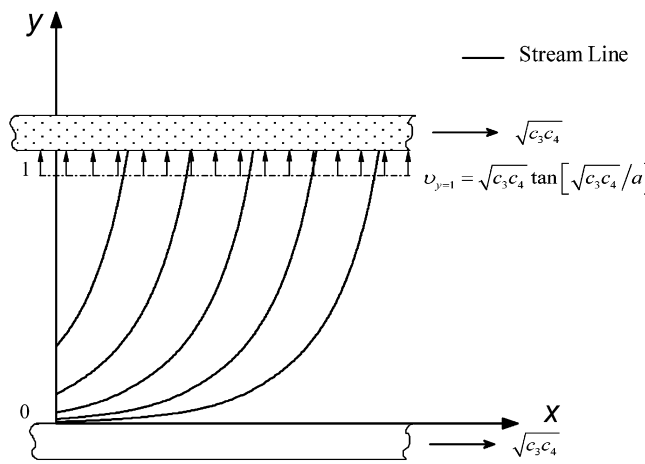

Fig. 7. The stream lines of Eqs. (66)–(69) with $c_5 = 0$.

6.2. The second solution

If the constant c_3 in Eq. (63) is equal to zero, the solution of previous sub-paragraph is non-sense. The correct solution has to be re-derived from Eq. (63) with the same procedure as that in the previous paragraph. The final expressions are

$$\theta = \theta_0 - a\ln[(x - c_6)(y - c_5)]/c_4, \tag{70}$$

$$u = -a/(x - c_6), \tag{71}$$

$$\upsilon = -a/(y - c_5), \tag{72}$$

$$p = p_0 + \rho a(2v - a)/2[1/(x - c_6)^2 + 1/(y - c_5)^2] \tag{73}$$

and

$$G = a[1/(x - c_6)^2 + 1/(y - c_5)^2] \tag{74}$$

The physical feature of Eqs. (70)–(74) is not clear and evident enough for viscous flow. However, it can still be a benchmark solution for the CHT of field synergy.

By the way, a special feature of this solution is that all the isobaric lines, the iso-mass source lines and the iso-velocity ($\sqrt{u^2 + \upsilon^2}$) lines are represented by the linear function $[1/(x - c_6)^2 + 1/(y - c_5)^2]$.

However, if considering nonviscous flow and $c_6 = 0 = c_7$, Eqs. (70)–(74) represent a linear parallel flow in a 45° inclined straight isometric solid path.

7. Analytical boundary synergy solution

It is probably easier to satisfy the boundary synergy (synergy only occurs along the boundary between fluid and its container) than full field synergy. However, the boundary is commonly important for the convection equipment. Therefore, an analytical solution is given for such case as an example.

Actually, a boundary synergy solution had been given some years ago by the first author [26]. It is a 2D laminar natural convection in a semi-infinite space with boundary suction along an infinite long vertical cold porous plate. Nevertheless, it was only mentioned a successful derivation of natural convection at that time, and has not yet been announced the solution is a boundary synergy one.

From [26], the solution is [where x direction is opposite to the gravity and the governing equation set is a little bit different from Eqs. (1)–(5)]:

$$G = 0, \tag{75}$$

$$u = -c_3\beta g a^2 \exp(-c_1 y/a)/[c_1^3(1 - v/a)] - c_4 v \exp(-c_1 y/v)/c_1 - c_2\beta g y/c_1 + c_5 \tag{76}$$

$$v = -c_1, \tag{77}$$

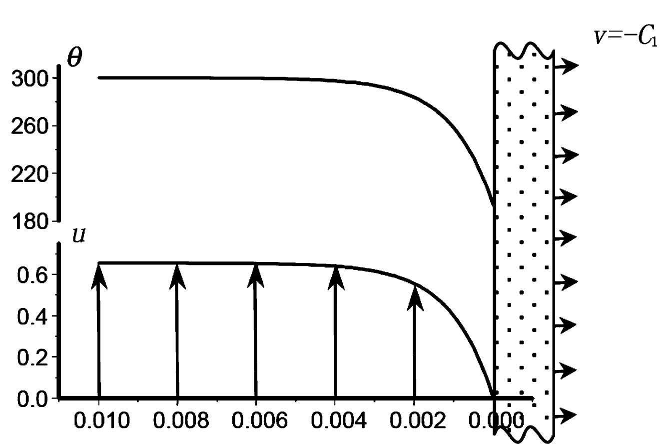

Fig. 8. A boundary field synergy case (Fig. 1 in [26]).

$$\theta = \theta_\infty + c_2 - a c_3 \exp(-c_1 y/a)/c_1, \tag{78}$$

$$q = 0. \tag{79}$$

It is assumed $c_1 > 0$, $c_2 = 0$ and $c_5 = c_3\beta g a^2/[c_1^3(1 - \upsilon/a)] + c_4\upsilon/c_1$. For $y = 0$ there are $u = 0$, $\upsilon = -c_1$ and $\theta = \theta_\infty - c_3 a/c_1$. The field synergy is satisfied on the boundary. The physical description can be seen in Fig. 8.

With different values of constants in Eqs. (76)–(78), there are some other boundary field synergy cases given in [26]; please refer to it.

8. Summary

(1) A thermodynamically strict discussion is concisely given about the convection, it is proven that the convection is not a rigorous "heat" transfer but mainly internal energy transfer by the movement of particles.
(2) Based on the previous discussion of convection, the concept of field synergy – the best convection "heat" transfer is the case where the velocity vectors are always perpendicular to the isothermal surfaces – is easy to understand and its governing equation is easy to set up.
(3) For further theoretically developing the field synergy principle and researching the artificial measures to accomplish field synergy, different kinds of algebraically explicit analytical exact solutions are derived and given, including solutions with heat source, with mass source, full field synergy solutions and boundary field synergy solutions. The derivation approaches include different methods of separating variables.

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